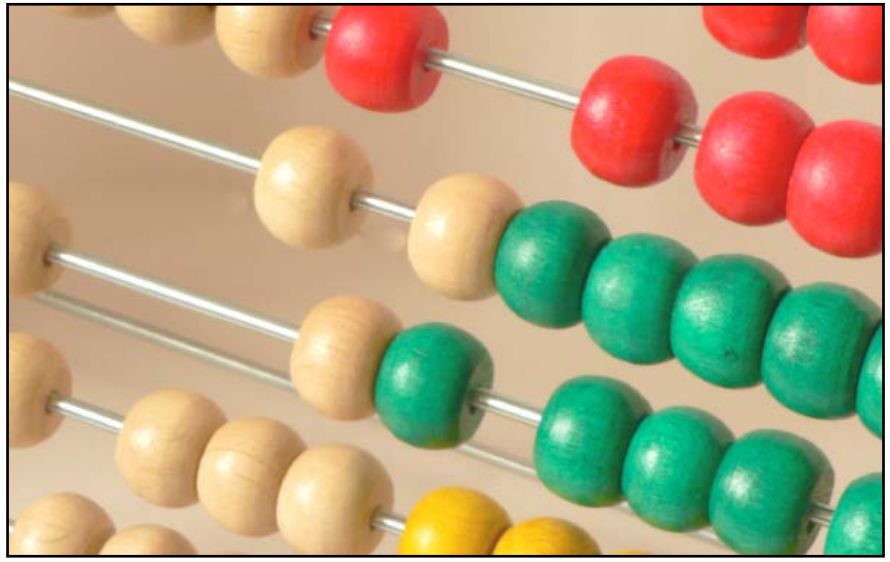
CALCULATIONS ("But that's not how we did it in school")



Fressingfield Primary
SHARE morning

Just to reflect …

- How did YOU do maths in school?
- Did you ENJOY it?
- Do you feel confident EXPLAINING maths homework to your children?
- If you found it negative in any way, might you transfer your own feelings to your children?

Just an aside …

I’ve heard a lot of people say they ‘were never any good at maths’ in school.

Do we go around saying, ‘I was never any good at reading in school’?

Maybe, as a society, it’s okay to find maths tricky?

Is this what we want with our own children?

Aims of today

1. To understand:

- **how** we teach calculation methods in school
- **why** we teach calculations in this way

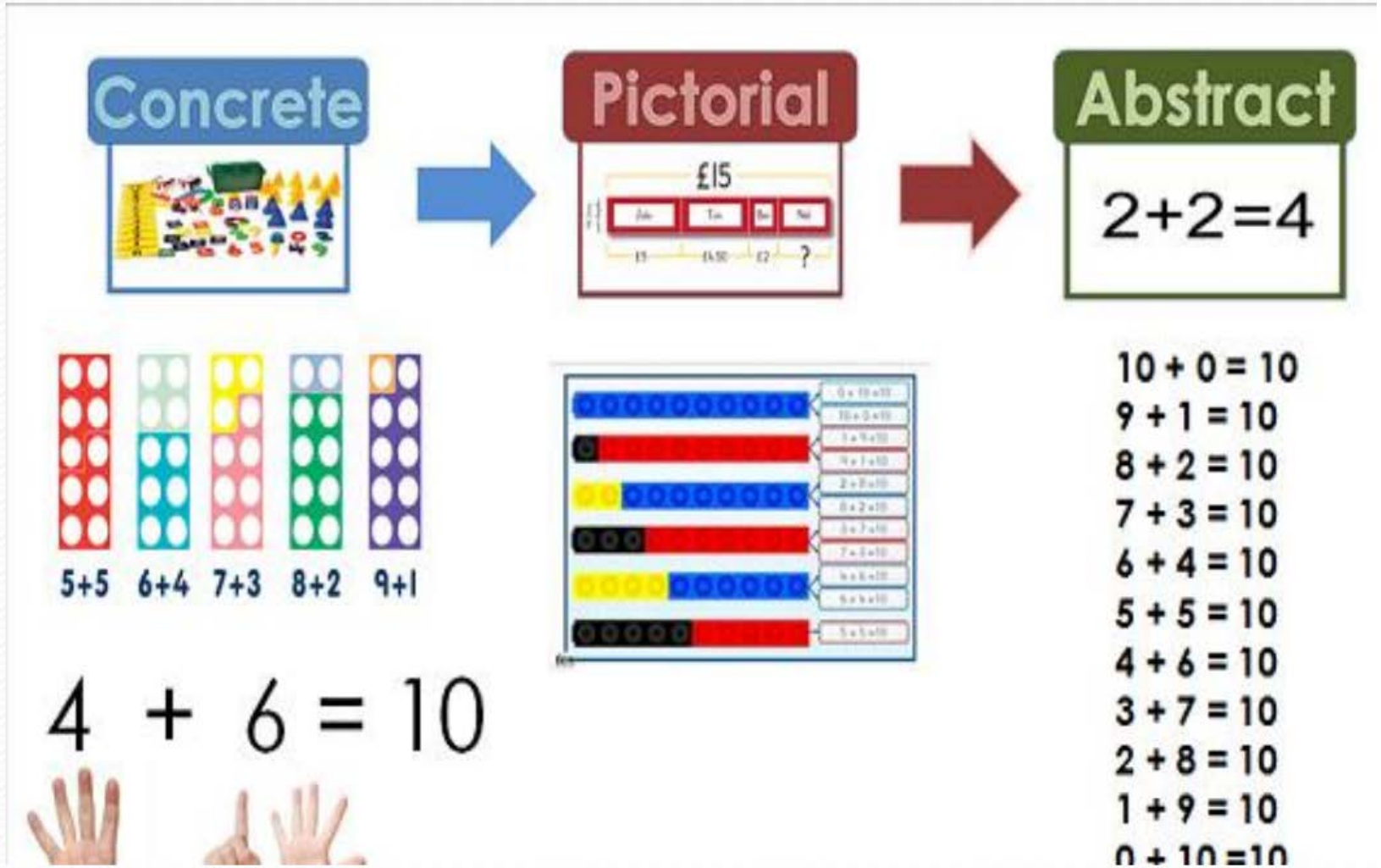
2. To give some tips about how to **help and support** your own children at home

What causes these negative feelings ?

- Not wanting to get things wrong (X)
- The feeling that you don't understand something
- If you don't understand the basics you do not have the knowledge to build upon

Compact methods that are familiar to us from our school days did not necessarily promote understanding of number.

Concrete, Pictorial, Abstract



How do we teach understanding?

Mental Strategies → **Informal Jottings** → **Expanded Methods** → **Compact Methods**

Mental Strategies

There is a far greater emphasis on teaching children mental strategies which are taught explicitly. Mental and oral work is part of every maths lesson including lots of counting, talking about numbers and using numbers in real life contexts

Informal Jottings

Children begin to record what they are doing with pictures and numbers. These recordings will help them understand what is happening and show how they have worked something out. They support the children's mental strategies.

Expanded Written Methods

These are the methods that may not be familiar to you. They "bridge" the children's development from working mentally to using the compact column methods.

These methods reflect the children's mental processes onto paper and support them when they begin to deal with larger numbers.

Compact Written Methods

These are the formal column sums that we all know and love !

Number lines

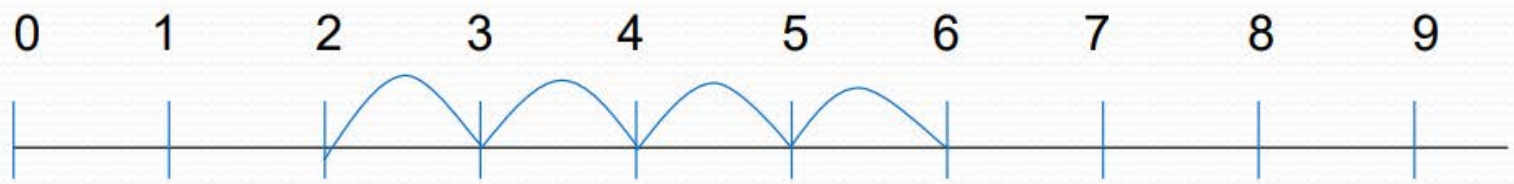
These are a very efficient way to calculate, as they are a **visual** support for children. A good maths teacher will use number lines right through past Year 6.

Children begin by using number tracks.
What is 2 more than 4 ?

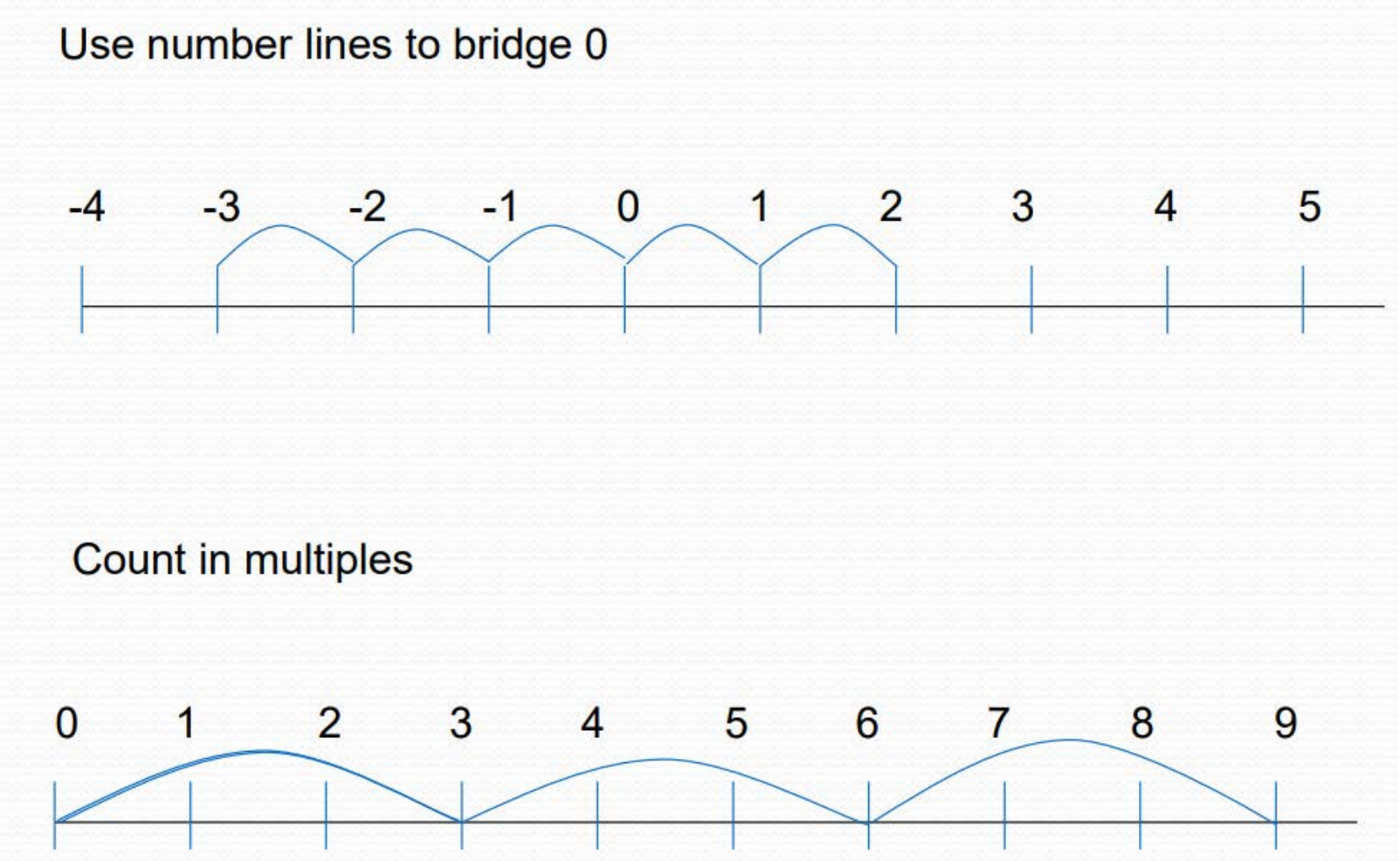
0	1	2	3	4	5	6	7	8	9

These can support children's counting on and counting back . Children can make "hops" along the number track, move counters or draw onto the tracks to help them.

This is then followed by a numbered number line.
Children can count on and count back.

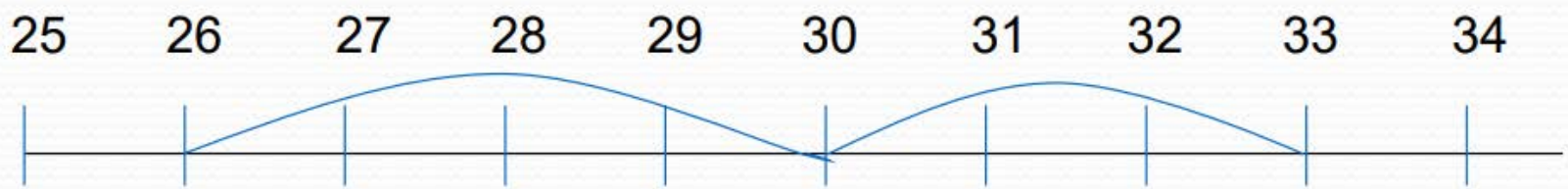


Progressing to **numbered** number lines …

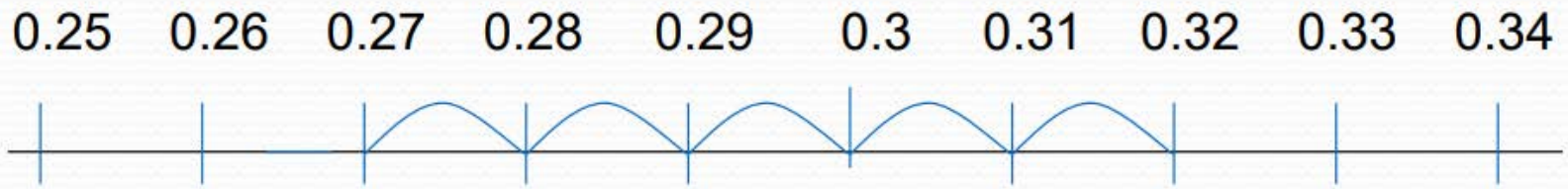


Using number lines for addition (including decimals) …

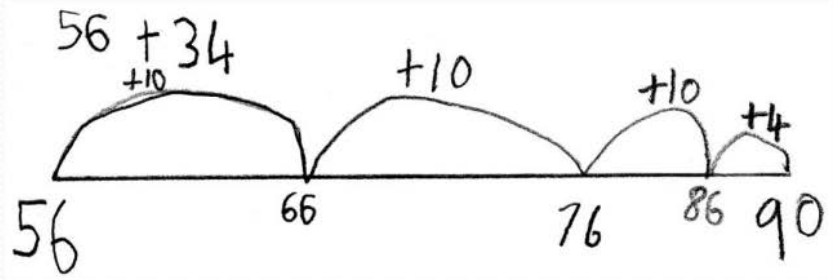
Calculate 26 + 7 (bridging a multiple of 10)



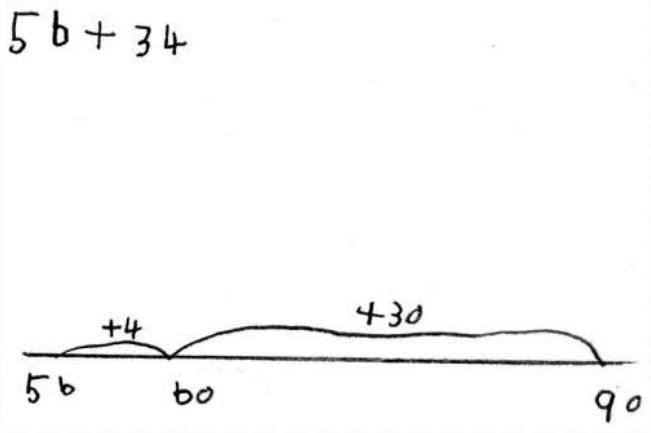
Calculate 0.27 + 0.05 =



See how we can use number lines to **add** and **subtract**. Notice the way we keep our **labelling** clear, with the **numbers at the bottom** and the **number operation** (what we are doing) above.



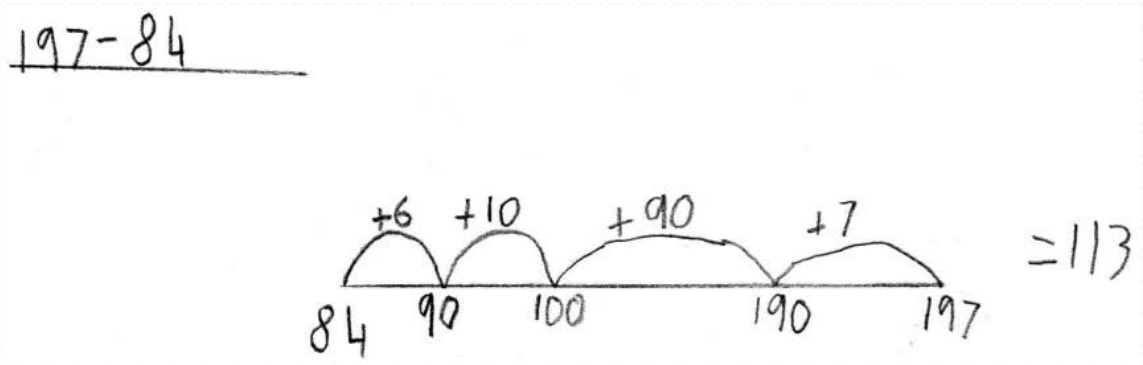
Here the child starts with 56 and has made 3 jumps of 10 and then a jump of 4 to add 34.



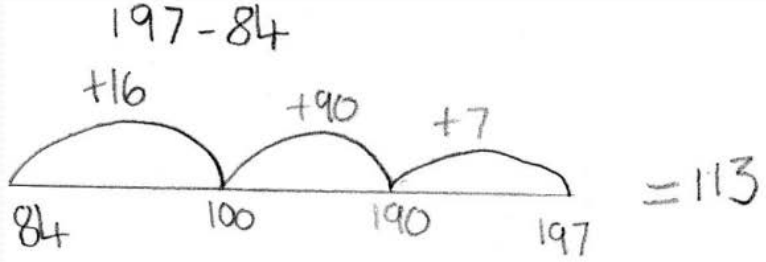
This child has made the jump of 4 to get to the multiple of 10 and then added the 30 in a single jump.

Number lines are brilliant for **connecting** addition and subtraction, by showing that **subtraction** can also be **counting on**.

- Number lines can help children gradually refine strategies.

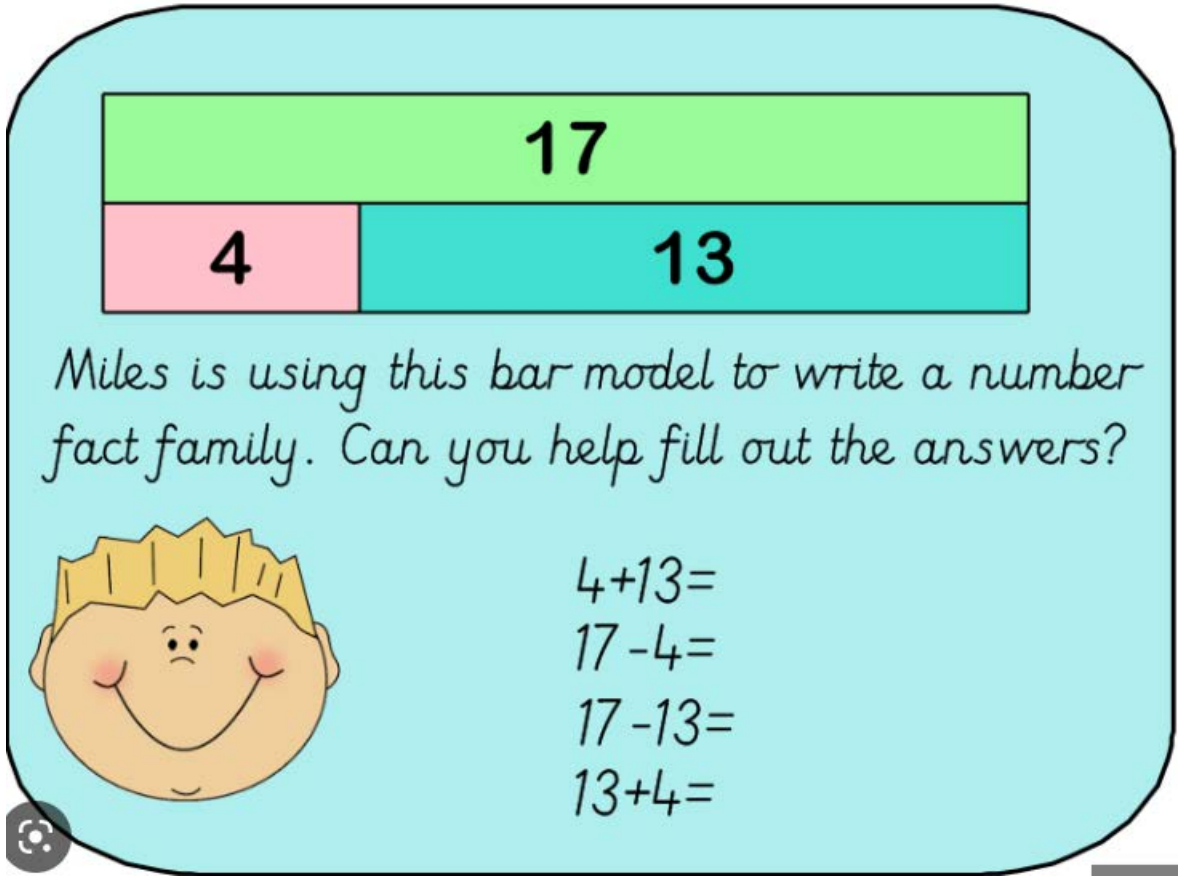


Child A has made 4 jumps to find the difference between 84 and 197.



Child B has made 3 jumps and can jump from 84 to 100 in a single jump

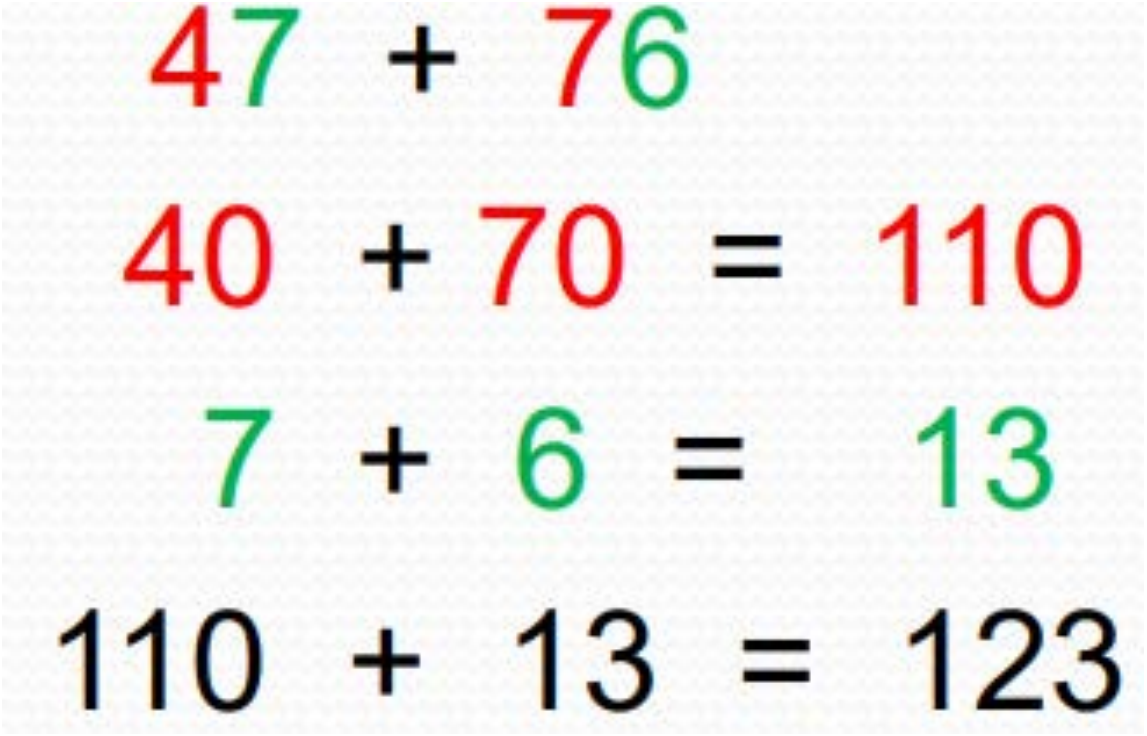
We also use **bar models**, which are another way of helping children to ‘see’ what maths they are doing.



Partitioning

This is a key mental strategy. It's learning to separate the **tens and ones (and hundreds, and thousands … and 1/10ths, 1/100ths, etc)**

Partitioning forms the basis of many of our **expanded methods.**



Expanded Method for Addition

Calculate 536 + 242

The children are taught to partition the number on paper in exactly the same way as they would do in their heads. They begin by adding the largest or most significant numbers first.

Stage 1

$$\begin{array}{r} 500 + 30 + 6 \\ +\ 200 + 40 + 2 \\ \hline 700 + 70 + 8 \end{array}$$

Answer 778

Stage 2

$$\begin{array}{r} 536 \\ +\ 242 \\ \hline 700 \\ 70 \\ \hline 8 \\ \hline 778 \end{array}$$

A little more about the **expanded method** …

Here, children are able to see the real value of all the numbers. We need to be very clear about the **language** we use, so that children understand **the size** of the numbers being added.

Stage 3

```
  2 7 8
+ 1 5 6
-------
    1 4   → "eight plus 6 equals fourteen"
  1 2 0   → "seventy plus fifty equals one hundred and twenty"
  3 0 0   → "two hundred plus one hundred equals three hundred"
-------
  4 3 4
```

*You may also notice that in this example the units are added first, in preparation for moving onto a compact method.

Calculation Methods

Addition

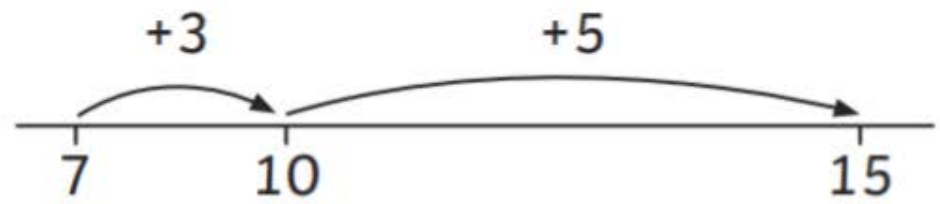
The different stages	Examples
Stage 1 Counting sets of objects	
Stage 2 Combining two sets of objects into one group and counting practically.	For 5 + 3 the children may get 5 objects, and then 3 more and count how many altogether.
Stage 3 Drawing dots - informal jottings. Then counting how many altogether.	3 + 5 = 8 • • • + • • • • •

Stage 5

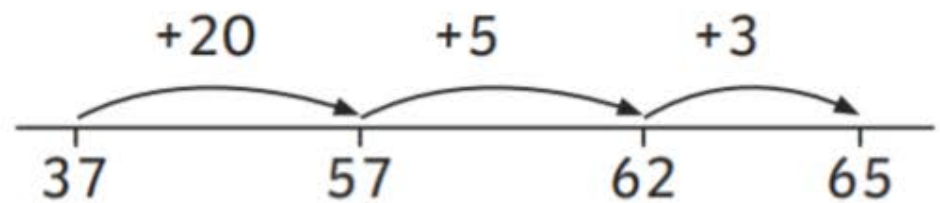
Steps in addition can be recorded on a number line. The steps often bridge through a multiple of 10.

1) Partition the smaller numbers into tens and ones.
2) Add on the tens.
3) Add on the ones.

7 + 8 = 15



37 + 28 = 65



Addition

The different stages	Examples
Stage 6 Partitioned numbers are then written under one another.	87 + 28 80 + 7 20 + 8 100 + 15 = 115
Stage 7 Write the numbers in columns Add the tens first.	87 + 28 100 15 115

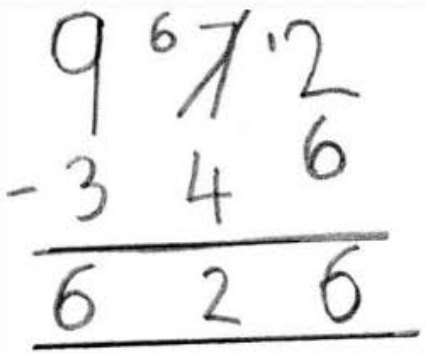
Adding the units first.	87 + 28 15 100 115
Stage 8 This then becomes the shorter method where numbers get carried into the next column.	87 + 28 115 1 1
Stage 9 Later, mover to adding three two digit numbers, two three digit numbers and numbers with amounts of digits.	249 + 96 345 1 1

Expanded Method for Subtraction

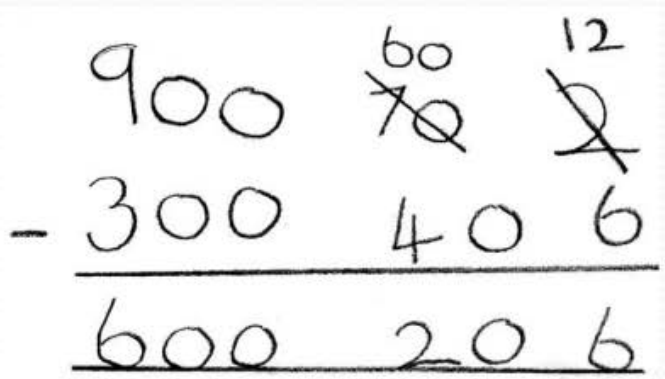
- The form of written subtraction sum with which we as adults are probably most familiar, is based upon a method called decomposition.
- Using the expanded method helps children to “see” and understand how the calculation works.

972 – 346

Compact Method



Expanded Method



Subtraction

The different stages	Examples
Stage 1 Practically get a group of objects together and then take some away.	
Stage 2 Jottings - draw a set of marks, and then cross some out.	12 - 5 = 7 
Stage 3 Count back on a number line with numbers already on it.	12 - 3 = 9 

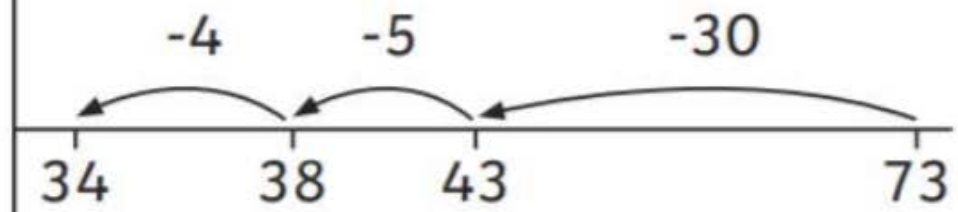
Stage 4

Using a number line.

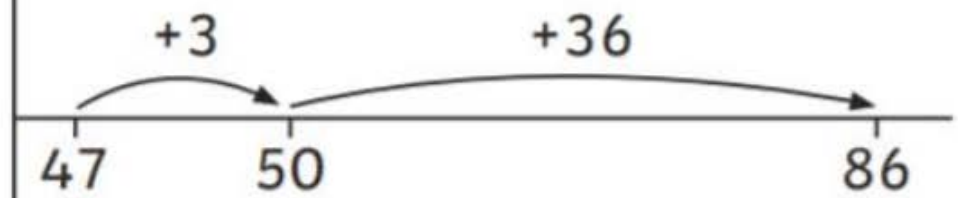
Work by counting back.

Also work out the difference by counting on.

73 - 39 = 34



Work out the difference between 47 and 86 = 39



The different stages	Examples
Stage 5 Partitioned numbers are written under one another. This is how we start introducing the column subtraction method.	77 - 25 = 70 + 7 -20 + 5 50 + 2 = 52
Stage 6 (Replace with 2 digit numbers) These show the two steps that lead to the shortened version of the column subtraction method. Always start with the units number.	73 - 26 = 70 + 3 -20 + 6 (split 77) 60 13 70 + 3 -20 + 6 40 + 7 6 13 73 -26 47

Stage 7

(Replace with 3 digit numbers)
These show the two steps that lead lead to the shortened version of the column subtraction method.
Always start with the units number.

652 - 475 =

$$\begin{array}{r} 600 + 50 + 2 \\ -400 + 70 + 5 \\ \hline \end{array}$$

$$\begin{array}{r} \overset{500}{\cancel{600}} + \overset{140}{\cancel{50}} + \overset{12}{\cancel{2}} \\ -400 + 70 + 5 \\ \hline 100 + 70 + 7 \end{array}$$

$$\begin{array}{r} \overset{5}{\cancel{6}}\ \overset{14}{\cancel{5}}\ \overset{12}{\cancel{2}} \\ -4\ 7\ 5 \\ \hline 1\ 7\ 7 \end{array}$$

Stage 8

(Replace with 4 digit numbers including 0)

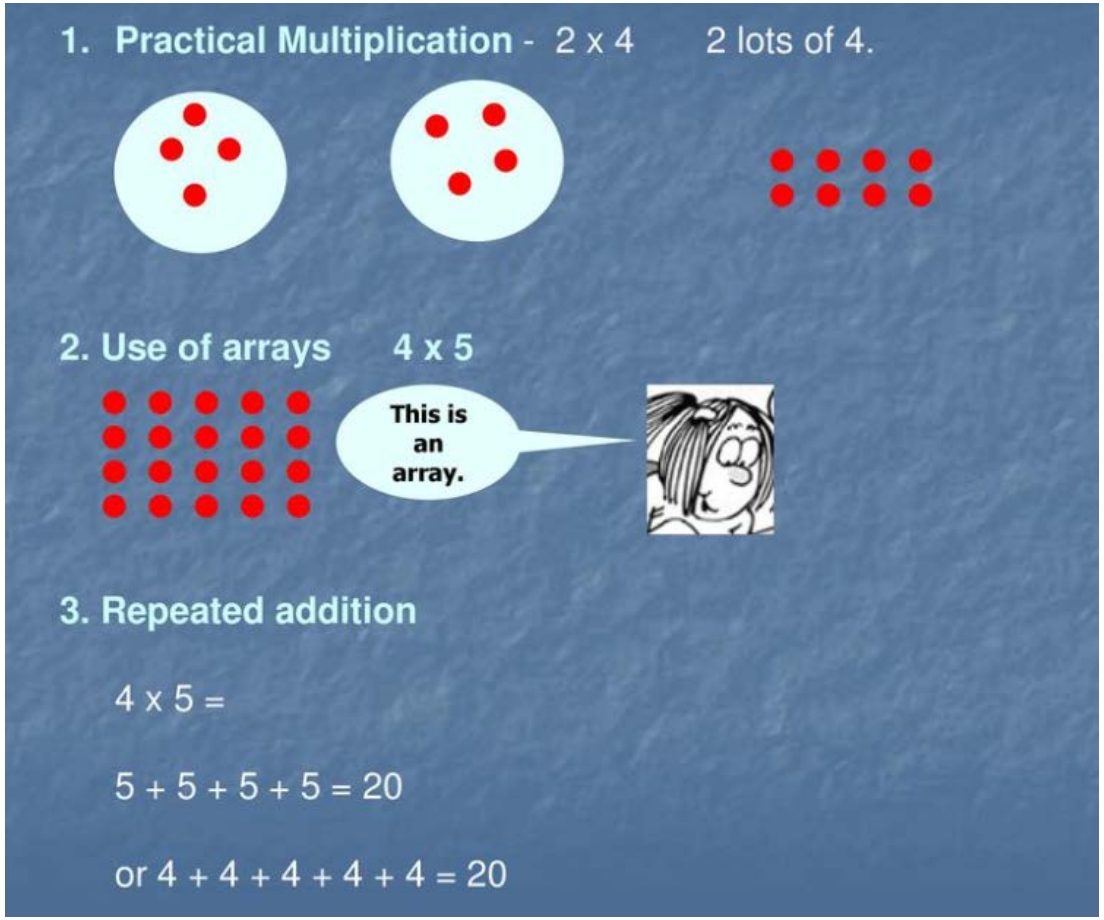
$$\begin{array}{r} \overset{5000}{\cancel{6000}} + \overset{900}{\cancel{000}} + \overset{100}{\cancel{00}} + 9 \\ -2000 + 100 + 20 + 3 \\ \hline 3000 + 800 + 80 + 6 \end{array} \rightarrow \begin{array}{r} \overset{5}{6}\ \overset{9}{0}\ \overset{10}{0}\ 9 \\ -2\ 1\ 2\ 3 \\ \hline 3\ 8\ 8\ 6 \end{array}$$

And multiplication … ?

It's really just **repeated addition**.

Have you ever heard of **arrays**?

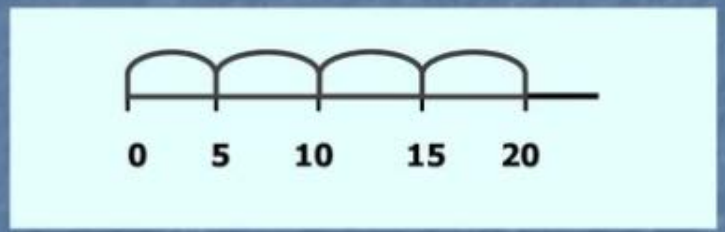
At first, we use plenty of plates and **real objects**. We can also use **number lines** (remember those?).



Multiplication cont .....

4. Repeated addition can also be done on a number line.

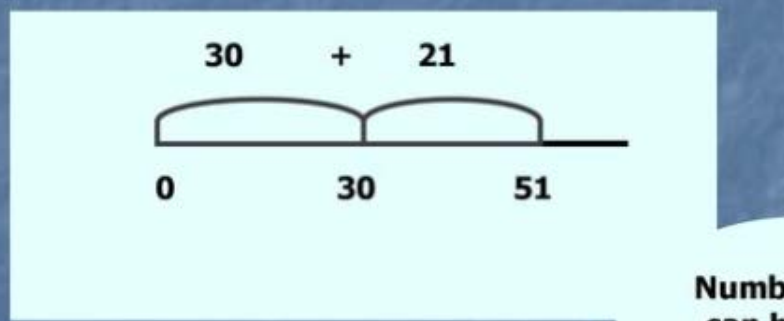
4 x 5



5. Partitioning – Simple recording

17 x 3 = (10 x 3) + (7 x 3)

30 + 21 = 51



Number lines can be used to do the addition part!

Multiplication cont .....

4. The Grid Method This is our key strategy for beginning to formally record multiplication. **17 x 3 = (10 x 3) + (7 x 3)**

X	10	7
3	**30**	**21**

30 + 21 = 51

Add the numbers inside the grid together to get the answer.

Try to add the numbers together mentally. If not, use a written method.

5. Multiplying two 2 digit numbers 18 x 23
Estimate 20 x 20 = 400.

X	10	8
20	**200**	**160**
3	**30**	**24**

200 + 160 + 30 + 24 = 360 + 54
360 + 54 = 414

```
  360
+  54
-----
    4
  110
  300
-----
  414
```

Multiplication cont .....

6. 3 digit by 2 digit 156 x 25 =
Estimate 160 x 20 = 3200

x	100	50	6
20	**2000**	**1000**	**120**
5	**500**	**250**	**30**

3120
+ 780
3900
1

7. 3 digit by 3 digit 152 x 385
Estimate 150 x 400 = 60000.

x	100	50	2
300	**30000**	**15000**	**600**
80	**8000**	**4000**	**160**
5	**500**	**250**	**10**

45 600
+ 12 160
760
58 520
1 1

Multiplication cont .....

8. Once children are confident with the grid method, they will be introduced to the following strategies for recording.

Short multiplication

17 x 3

```
   17
x   3
-----
   21  (7x3)
   30  (10 x 3)
-----
   51
```

leads to

```
   17
x   3
-----
   51
   2
```

9. Long multiplication 184 x 32

Estimate 180 x 30 = 5400.

```
   184
+   32
-------
   368  (184 x 2)
  5520  (184 x 30)
-------
  5888
```

And finally, **division**. Why do we put this last? Well, it's linked to **multiplication**. And we do seem to find it harder to **divide or share** than **multiply**. Note that, as we go through the school and teach division AND multiplication, the more tables the children know instantly, the better!

X	0	1	2	3	4	5	6	7	8	9	10	11	12
0	0	0	0	0	0	0	0	0	0	0	0	0	0
1	0	1	2	3	4	5	6	7	8	9	10	11	12
2	0	2	4	6	8	10	12	14	16	18	20	22	24
3	0	3	6	9	12	15	18	21	24	27	30	33	36
4	0	4	8	12	16	20	24	28	32	36	40	44	48
5	0	5	10	15	20	25	30	35	40	45	50	55	60
6	0	6	12	18	24	30	36	42	48	54	60	66	72
7	0	7	14	21	28	35	42	49	56	63	70	77	84
8	0	8	16	24	32	40	48	56	64	72	80	88	96
9	0	9	18	27	36	45	54	63	72	81	90	99	108
10	0	10	20	30	40	50	60	70	80	90	100	110	120
11	0	11	22	33	44	55	66	77	88	99	110	121	132
12	0	12	24	36	48	60	72	84	96	108	120	132	144

Division

1. **Sharing or Grouping** – Division is initially represented pictorially.

6 ÷2 = 3

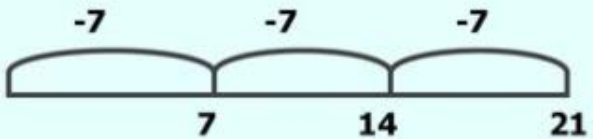
6 sweets **shared** between 2 people. How many each?

Sharing and grouping are two totally different concepts that children need to understand.

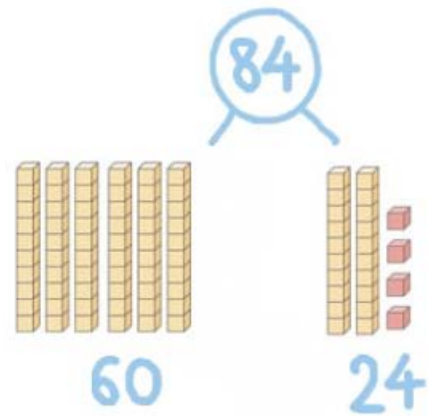
There are 6 people in a room. Put them into **groups** of 2. How many groups can you make?

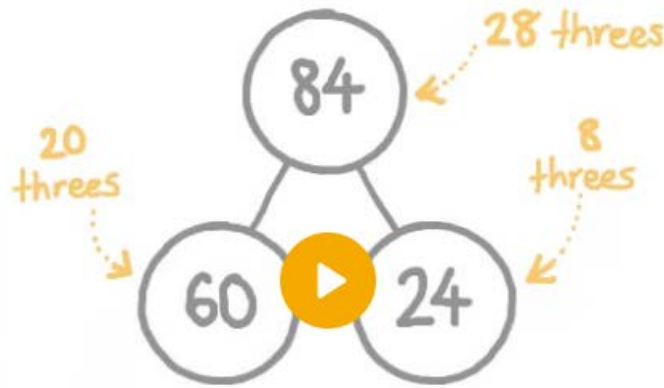
2. **Using a number line to show division.**

21 ÷7 = 3



Use the following part-whole model to find $84 \div 3$.





$30 \div 3 = 10$ (×2)

$60 \div 3 = 20$ (×2)

$20 + 8 = 28$

$24 \div 3 = 8$

$84 \div 3 = \boxed{28}$

Division cont .....

3. Using Multiples of the Divisor - Chunking.

90÷5 = 18

90
- 50 (10 x 5)

40
- 40 (8 x 5)

0

Start with 90 and take away multiples of 5.

4. Short division

87÷4 = 21 r 3

4 ⟌ 87

- 40 (10 x 4)

47

- 40 (10 x 4)

7

- 4 (1x4)

3

Division cont .....

5. Using Chunking with larger numbers.

875÷24 = 36 r 11

```
   4 | 875
 -     240  (10 x 24)
       ---
       635
 -     240  (10 x 24)
       ---
       395
 -     240  (10 x 24)
       ---
       155
 -     120  (5 x 24)
       ---
        35
 -      24  (1 x 24)
       ---
        11
```

6. Leading to sums using decimals.

The Golden Rules

- Let children work at their own pace and choose which methods they prefer.
- The expanded methods are efficient calculation strategies.
- Don’t rush into using formal compact methods.
- Always refer to a numbers true value when talking through calculations.
- Making mistakes is ok. Use them as learning opportunities.
- **ENJOY MATHS !**